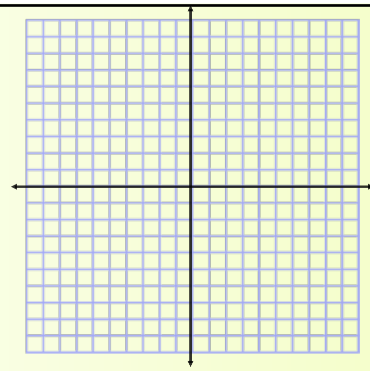
Precalc Warm Up # 4-4

1. Graph $y = x^2 - 4x + 9$



a. Is it a function?

b. Is it a 1-1 function?
(this means that not only does each input have one output, but each output has one input)

c. Restrict the domain so that it is as large as possible, and so that it is also a 1-1 increasing function.

EXERCISES 5.1

Questions: p. 113

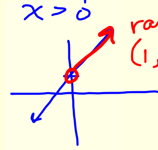
1. State the domain and range of the following relations.

(a) $\{(2,4), (3,-9), (-2,4), (3,9)\}$ → just 4 points. Domain is just the x-values of the points: $x \in \{-2, 2, 3\}$

2. Find the range for each of the following.

(a) $\{(x,y): y = x + 1, x \in \mathbb{R}^+\}$ → domain is positive reals. $x > 0$

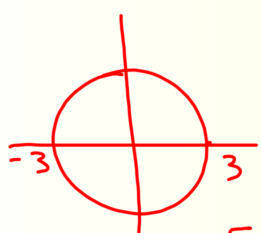
range: $(1, \infty)$



(c) $y = x^2 + 2x + 1, x > 2$

(e) $x^2 + y^2 = 9, -3 \le x \le 3$

(g) $y = x - 1, 0 < x \le 1$

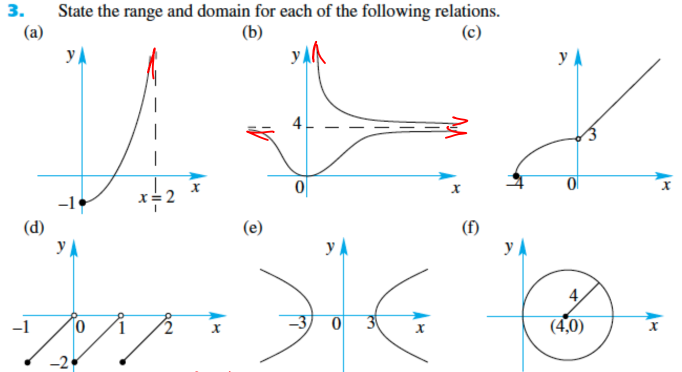


range: $[-3, 3]$

(i) $y = \sqrt{x}, x \geq 0$

(k) $y = \frac{4}{x+1}, x > 0$

3. State the range and domain for each of the following relations.



domain → smallest
x is −1, closed circle
so include it.

[−1, 2) open circle
at 2

5. Determine the implied doma

(a) $y = \frac{2x}{x+2}$

(d) $y = \sqrt{x^2 - 4}$

(g) $y = \frac{2}{x^3 + 1}$

(j) $x^2 - y^2 = a^2$ a ≥ 0

Next slide

dom: $x^3 + 1 \neq 0$

$x^3 \neq -1$

$x \neq -1$

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(d) $y = \sqrt{x^2 - 4}$

(g) $y = \frac{2}{x^3 + 1}$

(j) $x^2 - y^2 = a^2$ a ≥ 0

$\sqrt{a^2} = (\sqrt{a})^2$

Def.

$-y^2 = a^2 - x^2$

$y^2 = x^2 - a^2$

$y = \pm\sqrt{x^2 - a^2}$

domain:

$x^2 - a^2 \geq 0$

$x^2 \geq a^2$

$\sqrt{x^2} \geq \sqrt{a^2}$

$|x| \geq a$

Not $-a$ because it must be positive.

x must be more than (or = to) a units away from zero.

x x

$-a$ 0 a

$(-\infty, -a] \cup [a, \infty)$

$x \leq -a$ or $x \geq a$

6. Find the range of the following relations.

(a) $y = x - a, x < 0, a > 0$

(c) $y = a^2x - ax^2, x \ge \frac{1}{2}a, a > 0$

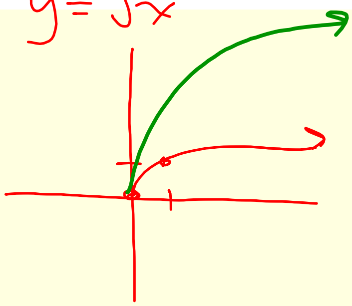
(e) $y = \frac{a}{x} + a, a > 0$

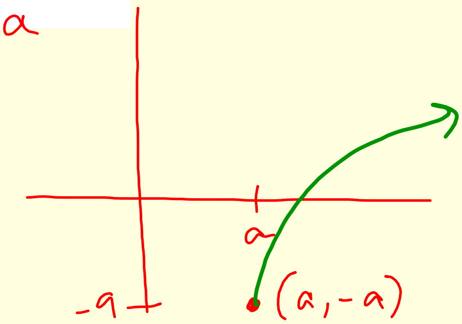
(g) $y = 2\sqrt{x - a} - a, a > 0$

down a

Rt a

$y = \sqrt{x}$





6. Find the range of the following relations.

(a) $y = x - a, x < 0, a > 0$

(c) $y = a^2x - ax^2, x \ge \frac{1}{2}a, a > 0$

(e) $y = \frac{a}{x} + a, a > 0$

(g) $y = 2\sqrt{x - a} - a, a > 0$

parabola, opens down

$y = -ax^2 + a^2x$

vertex: $x = -\frac{b}{2a} = \frac{-a^2}{2(-a)} = \frac{a}{2}$

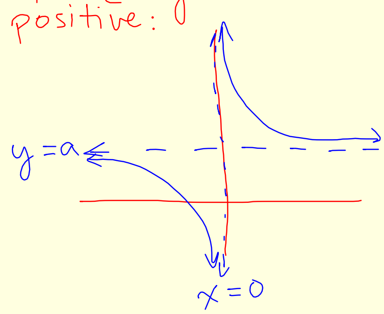
$y = -a\left(\frac{a}{2}\right)^2 + a^2\left(\frac{a}{2}\right) \dots$

hyperbola up "a"

vertical asymptote @ $x = 0$

horizontal @ $y = a$

a is positive:



Range: $y \ne a$

For the mapping $\underbrace{x \Rightarrow 3 + x^2}_{f(x) = 3 + x^2}$, $x \in \mathbb{R}$

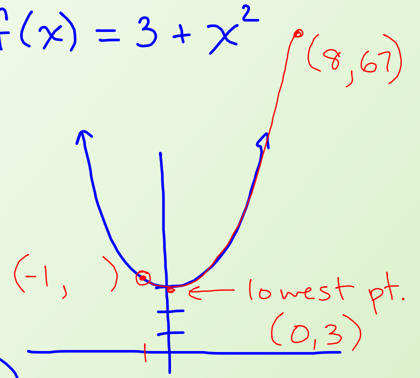
find a if f(a) = 10

$f(a) = 3 + a^2$

$10 = 3 + a^2$

$a^2 = 7$

$a = \pm\sqrt{7}$



Find the range if $x \in (-1, 8]$

Domain Restriction

$3 \leq y \leq 67$

Consider the function f: $\mathbb{R} \Rightarrow \mathbb{R}$, where $f(x) = 2 - x^2$

find

$-7 = 2 - (\quad)^2$

a. f(5) b. {x: f(x) = -7} c. {x: f(1 - x) = -7 }

input

$f(5) = 2 - 5^2$

$f(5) = -23$

$x^2 = 9$

$x = \pm 3$

$-7 = 2 - (1-x)^2$

$+9 = +(1 - 2x + x^2)$

$0 = x^2 - 2x - 8$

$= (x-4)(x+2)$

$\boxed{x = 4, -2}$

$x \in \{4, -2\}$

d. f(x + h) - f(x)

$2 - (x+h)^2 - (2 - x^2)$

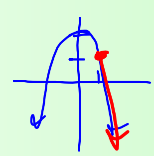
$\cancel{2} - (x^2 + 2xh + h^2) \cancel{-2} + \cancel{x^2}$

$\cancel{-x^2} - 2xh - h^2 \cancel{+x^2}$

e. find the range of f if x ∈ [1 , ∞)

$f(1) = 2 - (1)^2$

$= 1$



range: $(-\infty, 1]$

Consider $x \Rightarrow 4 - 2x^2$, $x \in \mathbb{R}$ $f(x) = 4 - 2x^2$

input

Find c given that f(c) = 3 outcome

$3 = 4 - 2(c)^2$

$-1 = -2c^2$

$\pm\sqrt{\frac{1}{2}} = \sqrt{c^2} \longrightarrow \boxed{c = \pm\frac{\sqrt{2}}{2}}$

Find b given that f(b) = b

input = b

outcome = b

$b = 4 - 2(b)^2$

$2b^2 + b - 4 = 0$

$b = \frac{-1 \pm \sqrt{\overbrace{1^2 - 4(2)(-4)}^{1+32}}}{4}$

$= \frac{-1 \pm \sqrt{33}}{4}$

HW: SL Book p. 120

#1-6, 9, 10, 12ab, 13a